27-Jan L1

Monday, January 27, 2025 11:46 AM

Things to Know:

- course documents are in Brightspace
- purchase textbook - in any format
- submit homeworks via **Gradescope** starting in Week 2

Section 1.1 Linear Systems

recall this basic 2-variable scenario...

ex. Admission tickets are sold to the county fair.
Adult tickets cost $5.
Kid tickets cost $2.

400 tickets are sold & revenue total is $1100.

How many of each ticket were sold?

let x = # adult tickets sold
let y = # kid tickets sold

2 variables $\Rightarrow$ (implies) 2 linear equations

$$\begin{aligned} x + y &= 400 \\ 5x + 2y &= 1100 \end{aligned}$$

$-2(x + y = 400)$

$-2x - 2y = -800$ (add these eq'ns)

$3x = 300$

$x = 100$

$\therefore 100 + y = 400$

$y = 300$

Solution: $(100, 300)$ ordered pair

to solve, we can solve 1st eq'n for y then substitute into 2nd eq'n

or multiply 1st eq'n by -2 then add 2 eq'ns together

Solving for 3 Variables

ex. rice yield with 3 different grades (quality level): • inferior • medium • superior

Given:

# inferior bundles (x)	# medium bundles (y)	# superior bundles (z)	total yield (lbs)
1	2	3	39
1	3	2	34
3	2	1	26

Create a linear system based on given data above

$x + 2y + 3z = 39$

Create a linear system based on given data above

$$\begin{array}{l} \text{I} \\ \text{II} \\ \text{III} \end{array} \left| \begin{array}{rrrr} x & +2y & +3z & = 39 \\ x & +3y & +2z & = 34 \\ 3x & +2y & +z & = 26 \end{array} \right|$$

want to simplify system s.t. $\left| \begin{array}{rrr} x & & = \\ & y & = \\ & z & = \end{array} \right|$ "solved"

again, use known tools to solve system / Solve for all 3 variables

start II − I

$$\begin{array}{l} \text{I} \\ \text{II}-\text{I} \\ \text{III} \end{array} \left| \begin{array}{rrrr} x & +2y & +3z & = 39 \\ & y & -z & = -5 \\ 3x & +2y & +z & = 26 \end{array} \right|$$

$$\left| \begin{array}{rrrr} x & +2y & +3z & = 39 \\ & y & -z & = -5 \\ & -4y & -8z & = -91 \end{array} \right|$$

$$\left| \begin{array}{rrrr} x & & +5z & = 49 \\ & y & -z & = -5 \\ & -4y & -8z & = -91 \end{array} \right|$$

$$\left| \begin{array}{rrrr} x & & +5z & = 49 \\ & y & -z & = -5 \\ & & -12z & = -111 \end{array} \right| \quad * -\frac{1}{12}$$

$$\left| \begin{array}{rrrr} x & & +5z & = 49 \\ & y & -z & = -5 \\ & & z & = 9.25 \end{array} \right|$$

$$\left| \begin{array}{rrrr} x & & +5z & = 49 \\ & y & & = 4.25 \\ & & z & = 9.25 \end{array} \right| \quad *(-5)$$

$$\left| \begin{array}{rrrr} & & & \\ & & & \\ & & & -0.25 \end{array} \right|$$

$$\begin{array}{rrrr} x & +3y & +2z & = 34 \\ -x & -2y & -3z & = 39 \\ \hline & y & -z & = -5 \end{array}$$

$$\begin{array}{rrrr} -3\text{I}: & -3x & -6y & -9z = -117 \\ \text{III}: & 3x & +2y & +z = 26 \\ \hline & & -4y & -8z = -91 \end{array}$$

$*-2$

$$\begin{array}{rrrr} & -2y & +2z & = 10 \\ x & +2y & +3z & = 39 \\ \hline x & & +5z & = 49 \end{array} \leftarrow$$

$*4$

$$\begin{array}{rrrr} & 4y & -4z & = -20 \\ & -4y & -8z & = -91 \\ \hline & & -12z & = -111 \end{array}$$

$$z = \frac{111}{12} = 9.25$$

$$\begin{array}{rrrr} y & -z & = -5 \\ & z & = 9.25 \\ \hline & y & = 4.25 \end{array}$$

$$\begin{array}{rrrr} x & +5z & = 49 \\ & -5z & = 46.25 \\ \hline x & & = 2.75 \end{array}$$

$-5z = 46.25$

$x = 2.75$

$x = 2.75$

$y = 4.25$

$z = 9.25$

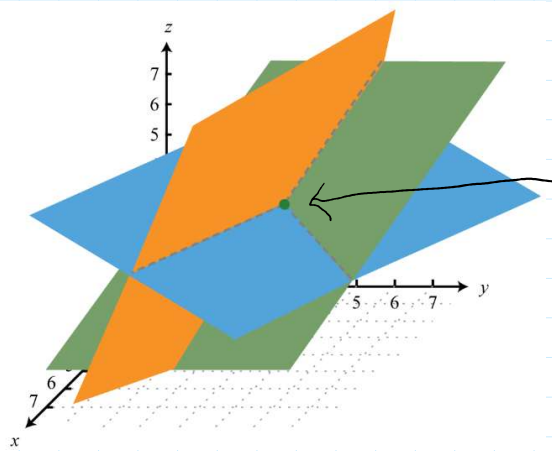
Solved

can write this solution in form of an ordered triple: (x, y, z)

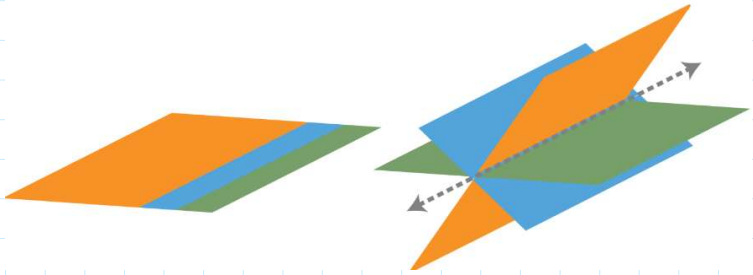
$$(2.75, 4.25, 9.25)$$

In general, there are 3 possible solution scenarios:

- single solution (ordered pair / triple)
  - ordered pair ↑ 2-variable system
  - triple ↑ 3-variable system
- infinitely many solutions
- no solutions

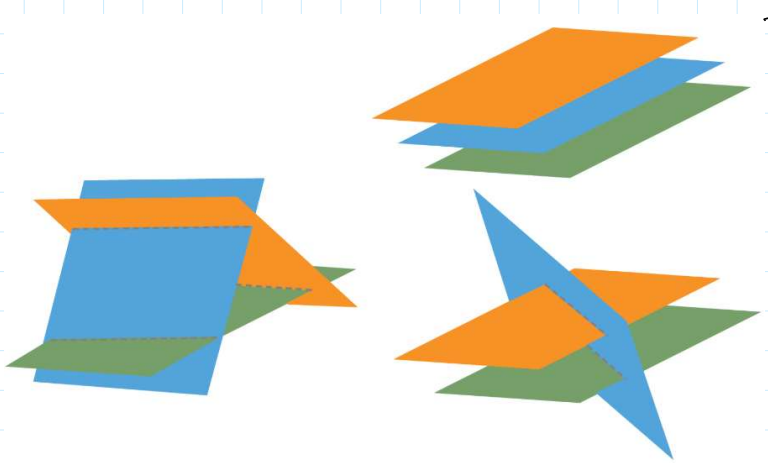


ordered triple (single solution)



infinitely many solutions

examples



no solutions

examples